

String Non(anti)commutativity for Neveu-Schwarz Boundary Conditions

Chandrasekhar Chatterjee · Sunandan Gangopadhyay ·
Arindam Ghosh Hazra · Saurav Samanta

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Abstract The appearance of non(anti)commutativity in superstring theory, satisfying the Neveu-Schwarz boundary conditions is discussed in this paper. Both an open free superstring and also one moving in a background antisymmetric tensor field are analyzed to illustrate the point that string non(anti)commutativity is a consequence of the nontrivial boundary conditions. The method used here is quite different from several other approaches where boundary conditions were treated as constraints. An interesting observation of this study is that, one requires that the bosonic sector satisfies Dirichlet boundary conditions at one end and Neumann at the other in the case of the bosonic variables X^μ being antiperiodic. The non(anti)commutative structures derived in this paper also leads to the closure of the super constraint algebra which is essential for the internal consistency of our analysis.

Keywords Non(anti)commutativity · Superstrings · Boundary conditions

1 Introduction

It is now a well established fact theoretically that at fundamental scale, space time becomes noncommutative (NC) in nature. This can be understood from a fundamental theory namely string theory where in the presence of a background antisymmetric $\mathcal{B}_{\mu\nu}$ field, the string end

C. Chatterjee · S. Gangopadhyay (✉) · S. Samanta
S.N. Bose National Centre for Basic Sciences, JD Block, Sector III, Salt Lake, Kolkata 700098, India
e-mail: sunandan@bose.res.in

C. Chatterjee
e-mail: chandra@bose.res.in

S. Samanta
e-mail: saurav@bose.res.in

A.G. Hazra
Department of Mathematics, Sundarban Mahavidyalaya Kakdwip, South 24 Parganas, West-Bengal,
India
e-mail: arindamg@bose.res.in

points become NC [1–3] and therefore the embedding D -brane coordinates also become NC. A detailed discussion of this problem in the context of bosonic string has been done in [4–6]. On the other hand noncommutativity for both the bosonic and the fermionic string theories has been explored in [7–9]. In [4, 7–9] the Neumann and Dirichlet boundary conditions (BC) which get mixed due to the presence of nonvanishing \mathcal{B} field were considered as Hamiltonian constraints. A different approach was followed in [6] where the constraints appear in an algebraic manner within the framework of Hamiltonian formulation. In [7] noncommutativity appears not only at the boundary but also in the bulk, where as in [10] noncommutativity was shown to exist only at the boundary. Andrade et al. [11] took the BC(s) as second class constraint and project the original coordinates on the constraint surface to recover a set of unconstrained string coordinates. Very recently the Faddeev-Jackiw symplectic formalism and a conformal field theoretic approach have been used to obtain the NC structure in [12–15].

However, there is an alternative method first initiated by Hanson, Regge and Teitelboim [16] for the free Nambu-Goto string and later extended in [17] for the Polyakov string, where instead of taking the BC(s) as constraints it is shown that the NC structure appears as a natural modification of the Poisson bracket (PB) to make it compatible with the BC(s). Previously in a couple of papers [18, 19], two of the authors used this approach to investigate this problem. Surprisingly a common point of all these studies in the superstring theory is that all the literatures are solely confined for the Ramond (R) BC(s) only. But as is well known in the case of fermionic string there is a choice between R BC(s) and Neveu Schwarz (NS) BC(s). This second type of BC is less studied in the research area. Here in this paper, we extend the above mentioned methodology to the superstring satisfying the NS BC(s). A nontrivial result we have found from the whole analysis is that contrary to the R case, bosonic sector of the superstring satisfies Dirichlet BC at one end and Neumann BC at the other end provided the bosonic variable X^μ is allowed to be antiperiodic. This observation is completely new and has not been discussed elsewhere. Further, the symplectic structure of the bosonic sector also keeps the superconstraint algebra involutive. The bracket structures have also been computed using the mode expansions of the bosonic and the fermionic coordinates.

The organization of the paper is as follows: In Sect. 2, the R-Neveu Schwarz (RNS) superstring action in the conformal gauge is briefly discussed to fix the notations. The section is then subdivided into two parts. In the first subsection, the BC(s) and the mode expansions of the fermionic sector of the superstring is given and the nonanticommutativity of the theory is revealed in the conventional Hamiltonian framework. In the next subsection, the PB structure and the BC(s) of the bosonic sector is discussed. In Sect. 3, we compute the super constraint algebra with the modified symplectic structure obtained in the previous section. The results obtained in Sect. 2 is further confirmed in Sect. 4 by the mode expansion method. This consistency check is performed separately for the bosonic and the fermionic sector. Section 5 discusses the non(anti)commutativity in the interacting superstring theory in the RNS formulation. Finally Sect. 6 is for conclusions.

2 RNS Free Superstring

In the first part of this section we briefly mention the canonical algebra of the basic fields of a free open superstring. Later we shall show how these algebraic structures get modified as a result of the boundary conditions of the theory. The action we take for our analysis is

given by [20, 21]¹

$$S = -\frac{1}{2} \int_{\Sigma} d^2\sigma (\eta_{\mu\nu} \partial_a X^\mu \partial^a X^\nu - i \bar{\psi}^\mu \rho^a \partial_a \psi_\mu). \tag{1}$$

The bosonic and the fermionic part of the above action can be separated out as

$$S = S_B + S_F \tag{2}$$

where,

$$S_B = -\frac{1}{2} \int_{\Sigma} d^2\sigma \eta_{\mu\nu} \partial_a X^\mu \partial^a X^\nu \quad \text{and} \quad S_F = \frac{1}{2} \int_{\Sigma} d^2\sigma i \bar{\psi}^\mu \rho^a \partial_a \psi_\mu. \tag{3}$$

The components of the Majorana spinor ψ are denoted as ψ_{\pm}

$$\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix}. \tag{4}$$

The Dirac antibracket of the first order action S_F is easily read off

$$\begin{aligned} \{\psi_+^\mu(\sigma), \psi_+^\nu(\sigma')\}_{D.B} &= \{\psi_-^\mu(\sigma), \psi_-^\nu(\sigma')\}_{D.B} = -i \eta^{\mu\nu} \delta(\sigma - \sigma'), \\ \{\psi_+^\mu(\sigma), \psi_-^\nu(\sigma')\}_{D.B} &= 0. \end{aligned} \tag{5}$$

On the other hand the action S_B gives the following brackets among the bosonic variables

$$\begin{aligned} \{X^\mu(\sigma), \Pi^\nu(\sigma')\} &= \eta^{\mu\nu} \delta(\sigma - \sigma'), \\ \{X^\mu(\sigma), X^\nu(\sigma')\} &= 0 = \{\Pi^\mu(\sigma), \Pi^\nu(\sigma')\} \end{aligned} \tag{6}$$

where Π_μ is the canonically conjugate momentum to X^μ , defined in the usual way. Equations (5) and (6) defines the preliminary symplectic structure of the theory. We shall now discuss the effects of BC(s) on these symplectic algebra for the fermionic and the bosonic sectors separately.

2.1 Fermionic Sector

Varying the fermionic part of the action (3)

$$\delta S_F = i \int_{\Sigma} d^2\sigma [\delta \bar{\psi}_\mu \rho^a \partial_a \psi^\mu - \partial_\sigma (\psi_-^\mu \delta \psi_{\mu-} - \psi_+^\mu \delta \psi_{\mu+})] \tag{7}$$

we obtain the Euler-Lagrange equation for the fermionic field

$$i \rho^a \partial_a \psi^\mu = 0 \tag{8}$$

¹We follow the conventions

$$\rho^0 = \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \rho^1 = i\sigma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

and take the induced world-sheet metric and target space-time metric as $\eta^{ab} = \{-, +\}$, $\eta^{\mu\nu} = \{-, +, +, \dots, +\}$ respectively.

together with the following BC(s):

$$\begin{aligned} \psi_+^\mu(0, \tau) &= \psi_-^\mu(0, \tau), \\ \psi_+^\mu(\pi, \tau) &= \lambda \psi_-^\mu(\pi, \tau) \end{aligned} \tag{9}$$

where $\lambda = \pm 1$ corresponds to the R BC(s) and the NS BC(s), respectively. The investigation involving the R BC(s) has been made in [13, 15, 18]. In this paper, we shall work with the NS BC(s) which we write in the following manner

$$(\psi_+^\mu(\sigma, \tau) - \psi_-^\mu(\sigma, \tau))|_{\sigma=0} = 0, \tag{10}$$

$$(\psi_+^\mu(\sigma, \tau) + \psi_-^\mu(\sigma, \tau))|_{\sigma=\pi} = 0. \tag{11}$$

Now the mode expansion of the components of Majorana fermion, satisfying the above set of BC(s) is given by [20, 21]:

$$\psi_-^\mu(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{n \in Z + \frac{1}{2}} d_n^\mu e^{-in(\tau - \sigma)}, \tag{12}$$

$$\psi_+^\mu(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{n \in Z + \frac{1}{2}} d_n^\mu e^{-in(\tau + \sigma)}.$$

From the above mode expansions it follows automatically that

$$\psi_-^\mu(-\sigma, \tau) = \psi_+^\mu(\sigma, \tau). \tag{13}$$

Furthermore, making use of (9), we obtain

$$\psi_\pm^\mu(\sigma = -\pi, \tau) = -\psi_\pm^\mu(\sigma = \pi, \tau), \tag{14}$$

$$\psi_\pm^\mu(\sigma = -2\pi, \tau) = \psi_\pm^\mu(\sigma = 2\pi, \tau)$$

in the NS-sector. Hence $\psi_\pm^\mu(\sigma, \tau)$ is an antiperiodic function of antiperiodicity 2π which naturally implies that it is a periodic function of periodicity 4π . We now essentially follow the methodology of [18] for the present case. First, we introduce the antiperiodic delta function $\delta_{(a)P}(x)$ of antiperiodicity 2π and periodicity 4π

$$\delta_{(a)P}(x) = -\delta_{(a)P}(x + 2\pi) = \frac{1}{4\pi} \sum_{n \in Z + \frac{1}{2}} e^{inx} \tag{15}$$

which satisfies the defining property of a periodic δ -function i.e.

$$\int_{-2\pi}^{2\pi} dx' \delta_{(a)P}(x' - x) f(x') = f(x) \tag{16}$$

where $f(x)$ is an arbitrary periodic function with periodicity 4π . Using this we write the following expression for ψ_-^μ and ψ_+^μ in the physical interval $[0, \pi]$ of the string

$$2 \int_0^\pi d\sigma' [\delta_{(a)P}(\sigma' + \sigma) \psi_+^\mu(\sigma') + \delta_{(a)P}(\sigma' - \sigma) \psi_-^\mu(\sigma')] = \psi_-^\mu(\sigma), \tag{17}$$

$$2 \int_0^\pi d\sigma' [\delta_{(a)P}(\sigma' + \sigma) \psi_-^\mu(\sigma') + \delta_{(a)P}(\sigma' - \sigma) \psi_+^\mu(\sigma')] = \psi_+^\mu(\sigma). \tag{18}$$

We define a matrix $\Lambda_{AB}(\sigma, \sigma')$

$$\Lambda_{AB}(\sigma, \sigma') = \begin{pmatrix} \delta_{(a)P}(\sigma' - \sigma) & \delta_{(a)P}(\sigma' + \sigma) \\ \delta_{(a)P}(\sigma' + \sigma) & \delta_{(a)P}(\sigma' - \sigma) \end{pmatrix} \tag{19}$$

to write (17) and (18) in a compact form

$$2 \int_0^\pi d\sigma' \Lambda_{AB}(\sigma, \sigma') \psi_B^\mu(\sigma') = \psi_A^\mu(\sigma); \quad (A, B = -, +). \tag{20}$$

From the above equation Λ can be interpreted as a matrix valued “delta function” which acts on the two component Majorana spinor. Instead of (5) we therefore propose the following antibrackets in the fermionic sector

$$\{\psi_A^\mu(\sigma), \psi_B^\nu(\sigma')\} = -2i \eta^{\mu\nu} \Lambda_{AB}(\sigma, \sigma'). \tag{21}$$

Making use of (19) we write this in its component form

$$\begin{aligned} \{\psi_+^\mu(\sigma), \psi_+^\nu(\sigma')\} &= \{\psi_-^\mu(\sigma), \psi_-^\nu(\sigma')\} = -2i \eta^{\mu\nu} \delta_{(a)P}(\sigma - \sigma'), \\ \{\psi_-^\mu(\sigma), \psi_+^\nu(\sigma')\} &= -2i \eta^{\mu\nu} \delta_{(a)P}(\sigma + \sigma'). \end{aligned} \tag{22}$$

Remarkably the above set of antibracket algebra is now completely consistent with the BC(s). To see this explicitly, we compute the anticommutator of $\psi_+^\nu(\sigma')$ with (10) and (11), the left hand side of which gives:

$$\begin{aligned} & -2i(\delta_{(a)P}(\sigma - \sigma') - \delta_{(a)P}(\sigma + \sigma'))|_{\sigma=0} \\ &= -2i \Delta_{-(a)}(\sigma, \sigma')|_{\sigma=0} = \frac{i}{\pi} \sum_{n \in Z + \frac{1}{2}} \sin(n\sigma) \sin(n\sigma')|_{\sigma=0} = 0, \end{aligned} \tag{23}$$

$$\begin{aligned} & -2i(\delta_{(a)P}(\sigma - \sigma') + \delta_{(a)P}(\sigma + \sigma'))|_{\sigma=\pi} \\ &= -2i \Delta_{+(a)}(\sigma, \sigma')|_{\sigma=\pi} = -\frac{i}{\pi} \sum_{n \in Z + \frac{1}{2}} \cos(n\sigma) \cos(n\sigma')|_{\sigma=\pi} = 0 \end{aligned} \tag{24}$$

where the form of the antiperiodic delta function (15) has been used. This completes the analysis of the fermionic algebra for the NS BC(s). In the next section we shall use these relations (22) to compute the super constraint algebra.

2.2 Bosonic Sector

Let us now study the bosonic sector of the superstring action (1). Varying the bosonic part of the action (1), we obtain the equation of motion for the bosonic field

$$(\partial_\sigma^2 - \partial_\tau^2)X^\mu = 0 \tag{25}$$

together with Dirichlet and Neumann BC(s)

$$\begin{aligned} \delta X^\mu|_{\sigma=0,\pi} &= 0, \\ X'^\mu|_{\sigma=0,\pi} &= 0. \end{aligned} \tag{26}$$

Now there are two cases depending on the periodicity of the bosonic variable X^μ . Usually, one is interested in theories with maximum Poincaré invariance and hence X^μ must be periodic (with a periodicity of 2π). This case has already been discussed in [18]. On the other hand antiperiodicity of X^μ is interesting because one encounters it for twisted strings on an orbifold [22]. In this paper we shall discuss this case in details.

We let the bosonic string coordinates $X^\mu(\sigma)$ to have a periodicity of 4π (antiperiodicity of 2π):²

$$X^\mu(\sigma + 4\pi) = X^\mu(\sigma). \tag{27}$$

Hence the integral (16) once again holds for the bosonic coordinate $X^\mu(\sigma)$. Restricting to the case of even(odd) functions $X^\mu_\pm(-\sigma) = \pm X^\mu_\pm(\sigma)$, it can be easily seen that (16) reduces to:

$$2 \int_0^\pi d\sigma' \Delta_{\pm(a)}(\sigma, \sigma') X^\mu_\pm(\sigma') = X^\mu_\pm(\sigma) \tag{28}$$

where $\Delta_{\pm(a)}$ were defined in (23) and (24). Therefore we propose the following equal time PB:

$$\{X^\mu(\tau, \sigma), \Pi_\nu(\tau, \sigma')\} = 2\delta^\mu_\nu \Delta_{\pm(a)}(\sigma, \sigma'). \tag{29}$$

It is now easy to observe that for $\Delta_{+(a)}(\sigma, \sigma')$ to appear in the above PB the end points must satisfy following BC(s)

$$\begin{aligned} X'^\mu(0) &= 0, \\ X^\mu(\pi) &= 0 \end{aligned} \tag{30}$$

and for $\Delta_{-(a)}(\sigma, \sigma')$, the appropriate BC(s) that the end points must satisfy read

$$\begin{aligned} X^\mu(0) &= 0, \\ X'^\mu(\pi) &= 0. \end{aligned} \tag{31}$$

We shall find in the next section that the symplectic structure of the bosonic sector also plays a crucial role in the closure of the super constraint algebra.

3 Super Constraint Algebra

In this section we shall compute the algebra of the super-Virasoro constraints using the modified symplectic structures derived in Sect. 2.

The complete set of super constraints are given by [18, 20, 21]:

$$\begin{aligned} \chi_1(\sigma) &= \Phi_1(\sigma) + \lambda_1(\sigma) \approx 0, \\ \chi_2(\sigma) &= \Phi_2(\sigma) + \lambda_2(\sigma) \approx 0 \end{aligned} \tag{32}$$

²Note that this is also in accord with the fermionic sector.

where,

$$\begin{aligned}
 \Phi_1(\sigma) &= (\Pi^2(\sigma) + (\partial_\sigma X(\sigma))^2), \\
 \Phi_2(\sigma) &= (\Pi(\sigma)\partial_\sigma X(\sigma)), \\
 \lambda_1(\sigma) &= -i\bar{\psi}^\mu(\sigma)\rho_1\partial_\sigma\psi_\mu(\sigma) = -i(\psi_-^\mu(\sigma)\partial_\sigma\psi_{\mu-}(\sigma) - \psi_+^\mu(\sigma)\partial_\sigma\psi_{\mu+}(\sigma)), \\
 \lambda_2(\sigma) &= -\frac{i}{2}\bar{\psi}^\mu(\sigma)\rho_0\partial_\sigma\psi_\mu(\sigma) = \frac{i}{2}(\psi_-^\mu(\sigma)\partial_\sigma\psi_{\mu-}(\sigma) + \psi_+^\mu(\sigma)\partial_\sigma\psi_{\mu+}(\sigma))
 \end{aligned}
 \tag{33}$$

and using the basic algebra of fermionic and bosonic variables (22, 29), we get the following algebra for super-Virasoro constraints:

$$\begin{aligned}
 \{\chi_1(\sigma), \chi_1(\sigma')\} &= 8(\chi_2(\sigma)\partial_\sigma\Delta_{+(a)}(\sigma, \sigma') + \chi_2(\sigma')\partial_\sigma\Delta_{-(a)}(\sigma, \sigma')), \\
 \{\chi_2(\sigma), \chi_2(\sigma')\} &= 2(\chi_2(\sigma')\partial_\sigma\Delta_{+(a)}(\sigma, \sigma') + \chi_2(\sigma)\partial_\sigma\Delta_{-(a)}(\sigma, \sigma')), \\
 \{\chi_2(\sigma), \chi_1(\sigma')\} &= 2(\chi_1(\sigma) + \chi_1(\sigma'))\partial_\sigma\Delta_{+(a)}(\sigma, \sigma').
 \end{aligned}
 \tag{34}$$

Apart from a numerical factor the above algebra has the same structure as in [18] with the only difference that $\delta_P(\sigma)$ occurring in [18] has been replaced by $\delta_{(a)P}(\sigma)$. Similarly one can show that the algebra of super currents

$$\begin{aligned}
 \tilde{J}_1(\sigma) &= 2J_{01}(\sigma) = \psi_-^\mu(\sigma)\Pi_\mu(\sigma) - \psi_-^\mu(\sigma)\partial_\sigma X_\mu, \\
 \tilde{J}_2(\sigma) &= 2J_{02}(\sigma) = \psi_+^\mu(\sigma)\Pi_\mu(\sigma) + \psi_+^\mu(\sigma)\partial_\sigma X_\mu
 \end{aligned}
 \tag{35}$$

among themselves and also with the super constraints (32) close. It is also interesting to note that both $\Delta_{+(a)}$ and $\Delta_{-(a)}$ appearing in the PB of the bosonic variables (29) gives the same constraint algebra (34). Furthermore, the closure of the algebra also indicates the internal consistency of our analysis.

4 Mode Expansions and Symplectic Algebra

In this section, we shall derive the fermionic algebra (22) and the bosonic algebra (29) from a mode expansion of the constituting fields. To do that we consider the mode expansions of the fermionic field (12). Here d_n^μ are Fourier modes and they satisfy the algebra

$$\{d_m^\mu, d_n^\nu\} = -\frac{i}{\pi}\eta^{\mu\nu}\delta_{m+n,0}.
 \tag{36}$$

This algebra can be obtained just by following the procedure of [12], in which they have computed the antibrackets among Fourier components of fermionic sector of superstrings (R sector) using Faddeev-Jackiw symplectic formalism [23]. This relation (36) between d 's can also be worked out from the contour argument [14, 22] and the operator product expansion.

The antibracket relations between $\psi_A^\mu(\sigma), \psi_B^\nu(\sigma')$ are then obtained by using (12) and (36)

$$\{\psi_-^\mu(\sigma), \psi_+^\nu(\sigma')\} = \frac{1}{2} \sum_{r,s \in Z + \frac{1}{2}} e^{-ir(\tau-\sigma)} e^{-is(\tau+\sigma)} \{d_r^\mu, d_s^\nu\}$$

$$\begin{aligned}
 &= -\frac{i}{2\pi} \eta^{\mu\nu} \sum_{r \in Z + \frac{1}{2}} e^{-ir(\tau-\sigma)} e^{ir(\tau+\sigma)} \\
 &= -2i \eta^{\mu\nu} \delta_{(a)P}(\sigma + \sigma').
 \end{aligned}
 \tag{37}$$

Proceeding exactly in the similar manner one can get back the other antibrackets of (22).

In order to study the bosonic sector, we first need the expressions of the mode expansion for the two different types of BC(s) (30) and (31).

For the first case (BC (30)) it is given by:

$$X^\mu(\tau, \sigma) = \sum_{n \in Z + \frac{1}{2}} \frac{\alpha_n^\mu}{n} e^{in\tau} \sin n\sigma
 \tag{38}$$

and for the other case (BC (31)) the mode expansion is

$$X^\mu(\tau, \sigma) = \sum_{n \in Z + \frac{1}{2}} \frac{\alpha_n^\mu}{n} e^{in\tau} \cos n\sigma.
 \tag{39}$$

The canonical momenta corresponding to (38) and (39) are given by

$$\begin{aligned}
 \Pi_\mu(\tau, \sigma) &= \eta_{\mu\nu} \partial_\tau X^\nu(\tau, \sigma) \\
 &= i \eta_{\mu\nu} \sum_{n \in Z + \frac{1}{2}} \alpha_n^\nu e^{in\tau} \sin n\sigma
 \end{aligned}
 \tag{40}$$

$$= i \eta_{\mu\nu} \sum_{n \in Z + \frac{1}{2}} \alpha_n^\nu e^{in\tau} \cos n\sigma.
 \tag{41}$$

Here also the algebra between the modes can be computed by following the methodology of [13, 23]:

$$\{\alpha_m^\mu, \alpha_n^\nu\} = -\frac{i}{\pi} \eta^{\mu\nu} m \delta_{m+n,0}.
 \tag{42}$$

Using (42) we obtain the same equal time PB given in (29).

5 The Interacting Theory

After finishing the analysis for the free theory, we shall now study the interacting case where a superstring moves in the presence of a constant antisymmetric tensor field $\mathcal{B}_{\mu\nu}$. The action given by [18, 24, 25]:

$$\begin{aligned}
 S &= \frac{-1}{2} \int_\Sigma d\tau d\sigma [\partial_a X^\mu \partial^a X_\mu + \epsilon^{ab} \mathcal{B}_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \\
 &\quad + i \psi_{\mu-} E^{\nu\mu} \partial_+ \psi_{\nu-} + i \psi_{\mu+} E^{\nu\mu} \partial_- \psi_{\nu+}]
 \end{aligned}
 \tag{43}$$

where, $\partial_+ = \partial_\tau + \partial_\sigma$, $\partial_- = \partial_\tau - \partial_\sigma$ and $E^{\mu\nu} = \eta^{\mu\nu} + \mathcal{B}^{\mu\nu}$. Now since the bosonic and fermionic sectors decouple, we can study them separately. Here we concentrate on the fermionic

sector. The variation of the fermionic part of the action (43) gives the classical equations of motion:

$$\partial_+ \psi_{v-} = 0, \quad \partial_- \psi_{v+} = 0 \tag{44}$$

and a boundary term that yields the following NS BC(s):³

$$\begin{aligned} E_{\nu\mu} \psi_+^{\nu} (0, \tau) &= E_{\mu\nu} \psi_-^{\nu} (0, \tau), \\ E_{\nu\mu} \psi_+^{\nu} (\pi, \tau) &= -E_{\mu\nu} \psi_-^{\nu} (\pi, \tau) \end{aligned} \tag{45}$$

at the endpoints $\sigma = 0$ and $\sigma = \pi$ of the string.

As in the free case, the above nontrivial BC(s) leads to a modification in the symplectic structure (5). The $\{\psi_{(\pm)}^{\mu}(\sigma, \tau), \psi_{\pm}^{\nu}(\sigma', \tau)\}$ is the same as (22). In the case of mixed bracket, we make the following ansatz:

$$\{\psi_+^{\mu}(\sigma, \tau), \psi_-^{\nu}(\sigma', \tau)\} = C^{\mu\nu} \delta_{(a)P}(\sigma + \sigma'). \tag{46}$$

Brackets $\psi_-^{\gamma}(\sigma')$ with the BC(s) (45) one obtains

$$E_{\nu\mu} C^{\nu\gamma} = -2i E_{\mu\gamma} \tag{47}$$

which on solving gives

$$C^{\mu\nu} = -2i[(1 - \mathcal{B}^2)^{-1}]^{\mu\rho} E_{\rho\gamma} E^{\gamma\nu}. \tag{48}$$

Above solution is written in a matrix notation as,

$$C = -2i[(1 - \mathcal{B}^2)^{-1}(1 + \mathcal{B})^2] \tag{49}$$

where $C = \{C^{\mu\nu}\}$. Thus we get the modified mixed bracket in the form

$$\{\psi_+^{\mu}(\sigma, \tau), \psi_-^{\nu}(\sigma', \tau)\} = -2i[(1 - \mathcal{B}^2)^{-1}]^{\mu\rho} E_{\rho\gamma} E^{\gamma\nu} \delta_{(a)P}(\sigma + \sigma'). \tag{50}$$

If we take the limit $\mathcal{B}_{\mu\nu} \rightarrow 0$ in the above equation we get back the last relation of (22).

6 Conclusions

In string theory the modification of Poisson algebra is a consequence of the nontrivial BC(s). In this paper, we have studied this problem for an open superstring satisfying the NS BC(s). Following the approach of [18], here also the domain of the string length is extended from $[0, \pi]$ to $[-\pi, \pi]$ to get the antiperiodic BC(s). This construction enables us to get the 2×2 matrix valued δ function in the algebra of the fermionic sector. Apart from a numerical factor the fermionic algebra is identical to the result obtained in [18]. In this sense this paper is an extension of [18], where only R BC(s) was used.

However for the bosonic part of the superstring the result of present paper is drastically different [18]. We stress that the symplectic algebra of the bosonic variables, in this paper contains both $\Delta_{+(a)}(\sigma, \sigma')$ and $\Delta_{-(a)}(\sigma, \sigma')$ which is completely different from the R

³The boundary term also leads to R BC(s). Detailed investigations involving R BC(s) has already been carried out in [18, 24, 25].

case where only $\Delta_+(\sigma, \sigma')$ was present. Interestingly this symplectic structure containing both $\Delta_{\pm(a)}(\sigma, \sigma')$, keeps the superconstraint algebra closed provided one imposes Neumann BC(s) at one end and Dirichlet BC(s) at the other end of the string in the bosonic sector. This observation is completely new and has not been noticed before in the literature.

Finally to complete the analysis, we have calculated the non(anti)commutative structures for the interacting case by employing the same procedure. As one expects, without the background field term the interacting results take the limiting value of the free case.

References

1. Seiberg, N., Witten, E.: J. High Energy Phys. **09**, 032 (1999)
2. Douglas, M.R., Nekrasov, N.A.: hep-th/0108158
3. Szabo, R.J.: hep-th/0109162
4. Chu, C.-S., Ho, P.-M.: Nucl. Phys. B **550**, 151 (1999)
5. Schomerus, V.: J. High Energy Phys. **06**, 030 (1999)
6. Zabzine, M.: J. High Energy Phys. **0010**, 042 (2000)
7. Ardalan, F., Arfaei, H., Sheikh-Jabbari, M.M.: J. High Energy Phys. **9902**, 016 (1999)
8. Chu, C.-S., Ho, P.-M.: Nucl. Phys. B **568**, 447 (2000)
9. Braga, N.R.F., Godinho, C.F.L.: Phys. Lett. B **570**, 111–117 (2003). hep-th/0306163
10. Kim, W.T., Oh, J.J.: Mod. Phys. Lett. A **15**, 1597 (2000)
11. De Andrade, M.A., Santos, M.A., Vancea, I.V.: J. High Energy Phys. **0106**, 026 (2001)
12. Jing, J.: Phys. Rev. D **73**, 086001 (2006)
13. Jing, J., Long, Z.W.: Phys. Rev. D **72**, 126002 (2005)
14. Chakraborty, B., Gangopadhyay, S., Hazra, A.G.: Phys. Rev. D **74**, 105011 (2006). hep-th/0608065
15. Gangopadhyay, S., Hazra, A.G.: Phys. Rev. D **75**, 065026 (2007). hep-th/0703091
16. Hanson, A.J., Regge, T., Teitelboim, C.: Constrained Hamiltonian System. Accademia Nazionale Dei Lincei, Roma (1976)
17. Banerjee, R., Chakraborty, B., Ghosh, S.: Phys. Lett. B **537**, 340–350 (2002). hep-th/0203199
18. Chakraborty, B., Gangopadhyay, S., Hazra, A.G., Scholtz, F.G.: Phys. Lett. B **625**, 302–312 (2005). hep-th/0508156
19. Gangopadhyay, S., Hazra, A.G., Saha, A.: Phys. Rev. D **125023** (2006). hep-th/0701012
20. Green, M.B., Schwarz, J.H., Witten, E.: Superstring Theory, vol. 1. Cambridge University Press, Cambridge (1987)
21. Lüst, D., Theisen, S.: Lectures on String Theory. Springer, Berlin (1989)
22. Polchinski, J.: String Theory, vol. II. Cambridge University Press, Cambridge (1998)
23. Faddeev, L., Jackiw, R.: Phys. Rev. Lett. **60**, 1692 (1988)
24. Haggi-Mani, P., Lindstrom, U., Zabzine, M.: Phys. Lett. B **483**, 443–450 (2000)
25. Lindstrom, U., Rocek, M., van Nieuwenhuizen, P.: Consistent boundary conditions for open strings. hep-th/0211266